CDI-II **Ficha 2** 8/3/21

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Compacto ≡ limitado e fechado.

$A \subset \mathbb{R}^n$ diz-se limitado se existir uma bola que contém o conjunto.

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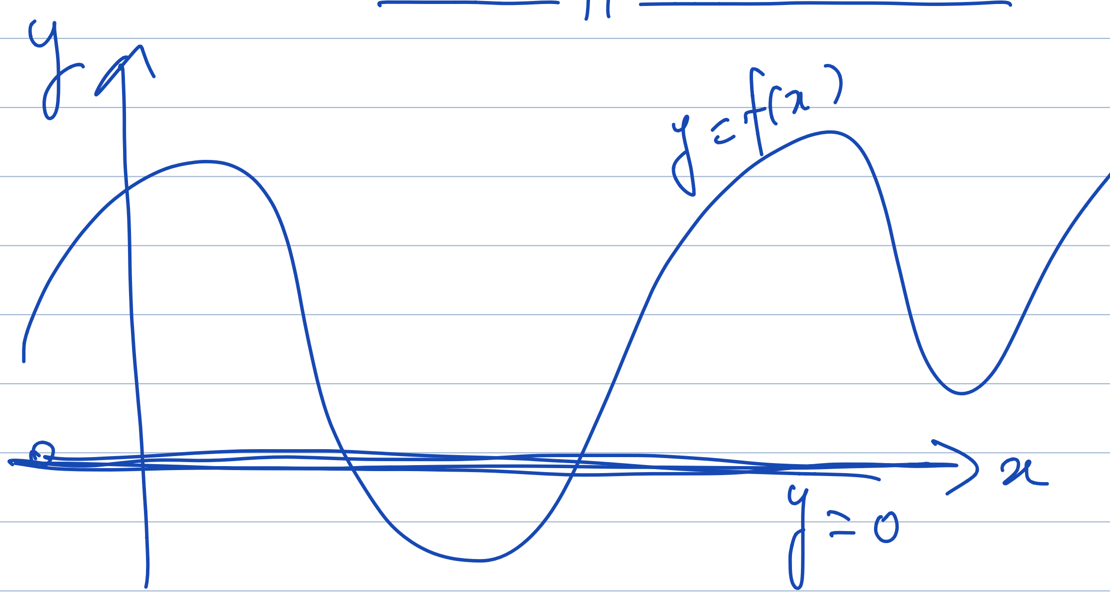
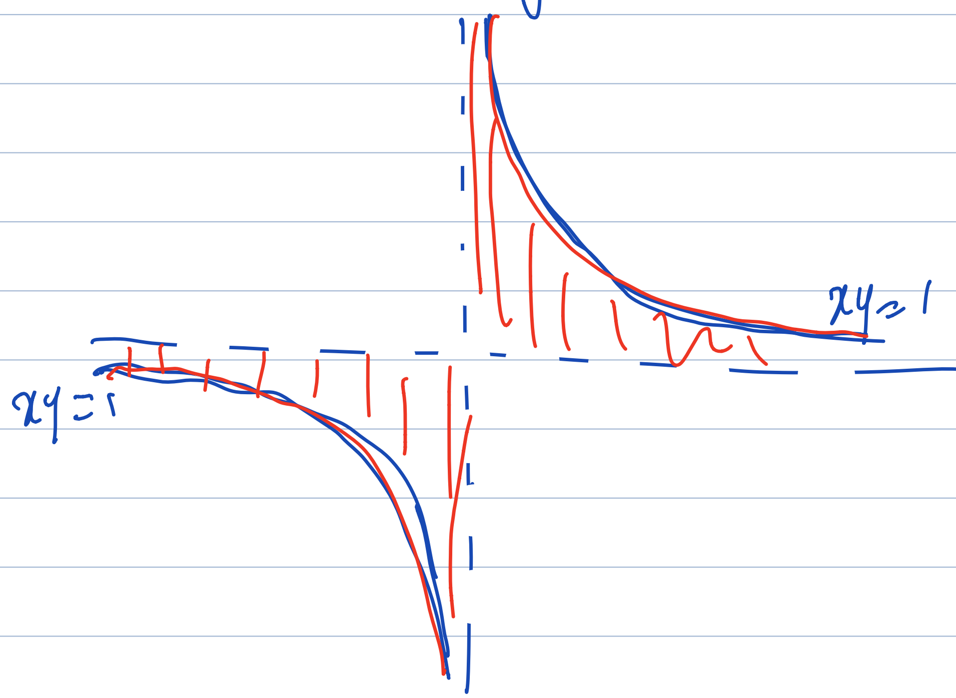


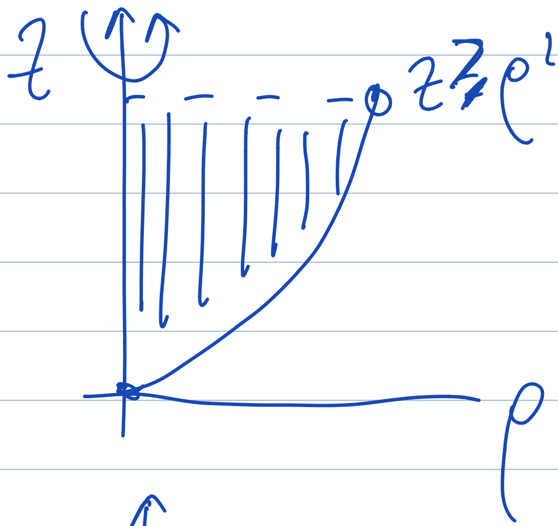
1-b) $\ln(xy) \leq 0$

$0 < xy \leq 1$

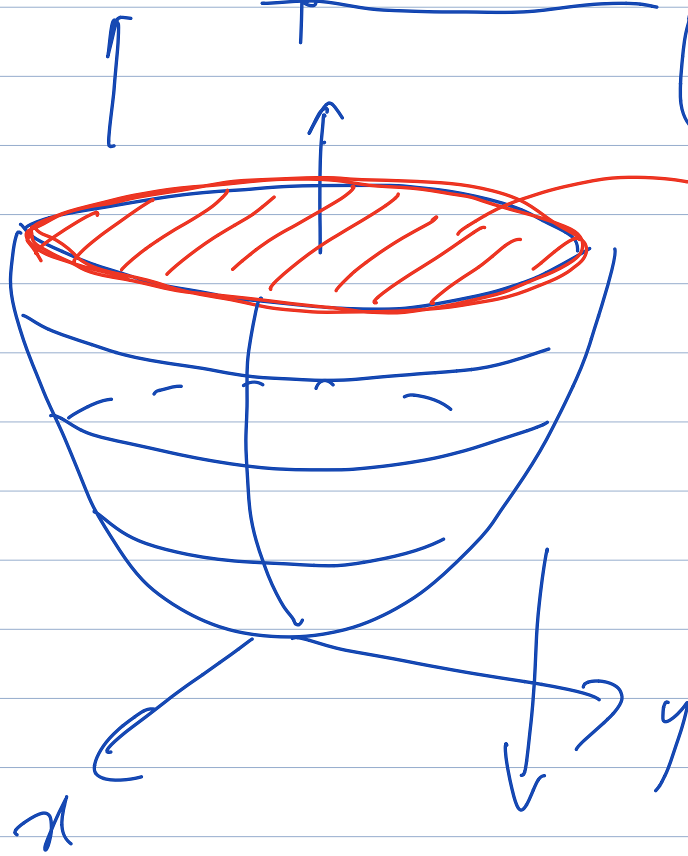


1-c) $x^2+y^2 \leq z < 1$

$r^2 \leq z < 1$



Sólido



$z = 1$

$\rho^2 \le z < 1$

$x^2+y^2 \le z < 1$

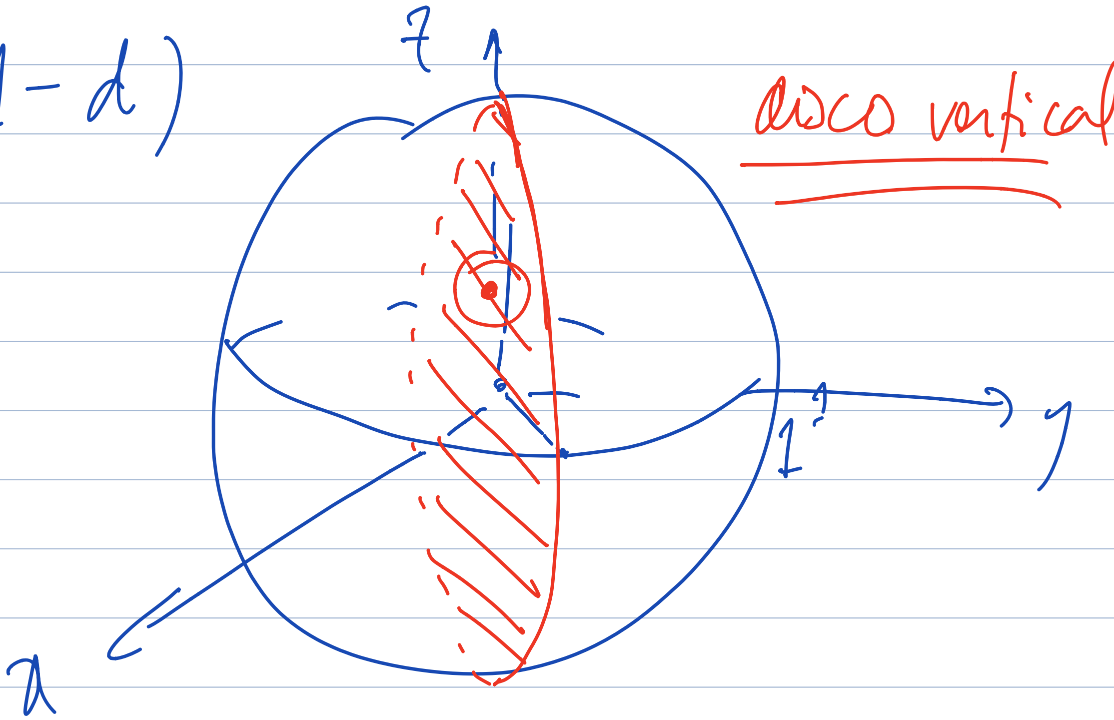
$x^2+y^2 < 1$

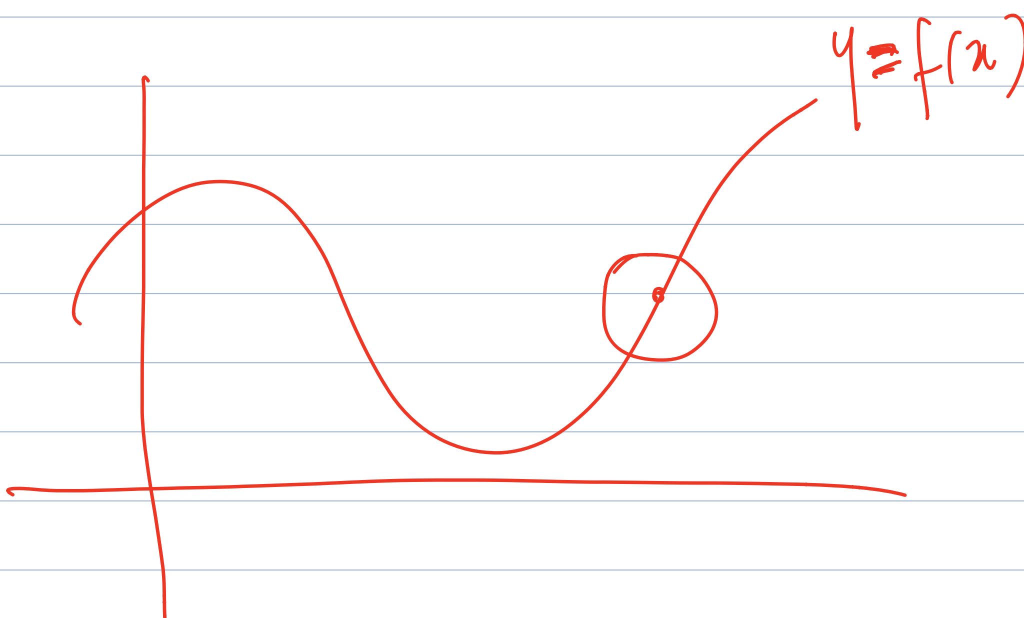
$0 < z^2 < 1$

$\boxed{x^2+y^2+z^2 < 2}$

$$\{x^2+y^2 = z \le 1\} \cup \{z = 1,\ x^2+y^2 \le 1\}$$

1 - d)

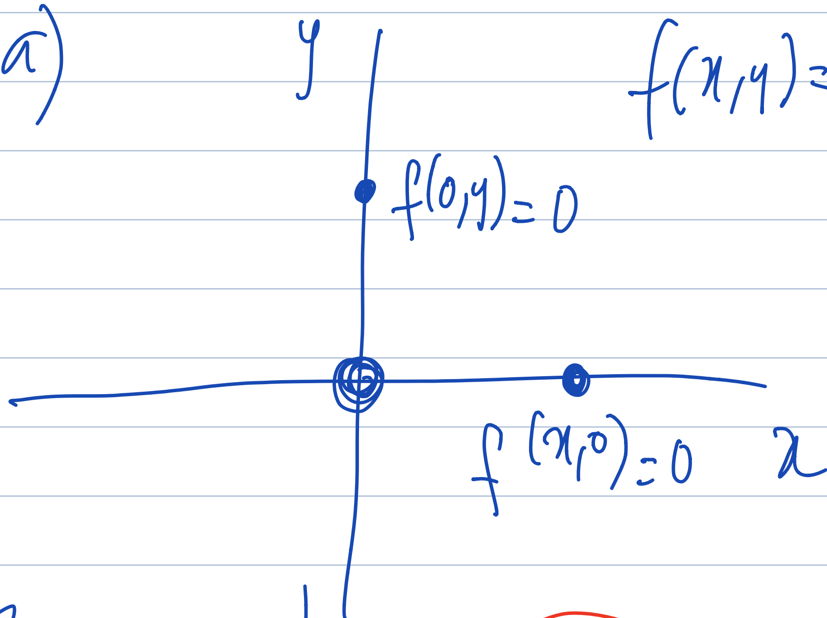




$$\bar{A} = \text{int}(A) \cup \text{front}(A)$$

2 - a)

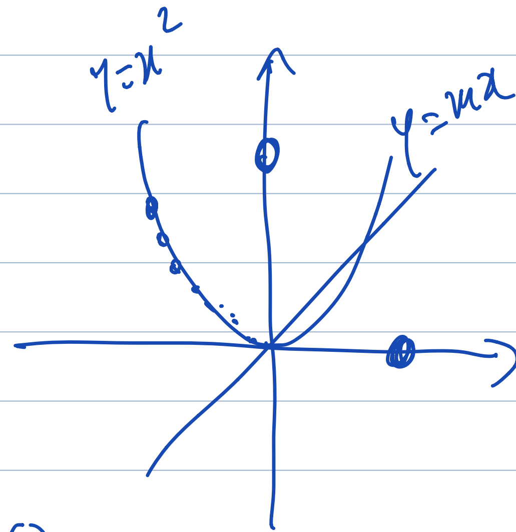
$$f(x,y) = \frac{x^2 y}{x^2+y^2}$$



$$\left| \frac{x^2 y}{x^2+y^2} - 0 \right| = \underbrace{\frac{x^2}{x^2+y^2}}_{\leq 1} |y| \leq |y| \to 0$$

$$\boxed{\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^2+y^2} = 0}$$

2-c) $\dfrac{x^2 y}{x^4 + y^2}$



$$\frac{x^2 x^2}{x^4 + x^4} = \frac{1}{2} \neq 0$$

dois candidatos a limite

$\Rightarrow$ o limite não existe.

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2- extra: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x y^2}{x^4 + y^2} = 0$

$$\left| \frac{x y^2}{x^4 + y^2} \right| \leq |x| \longrightarrow 0$$

≤ 1

2-d) $\left(\dfrac{x^2 y}{x^2+y^2}\right)\left(\dfrac{\ln(x^2+y^2)}{x^2+y^2}\right)$

↑ 2-a) ↑ notável

2-e) → dois candidatos.

2-f) $x\ln(xy) = x\ln x + \boxed{x\ln y}$

↑ notável

$y = x^k$

$x \ln x + x \ln(x^k) =$

$= x \ln x + k\, x \ln x \longrightarrow 0$

$\uparrow$

$y = e^{-\frac{1}{x^2}}$

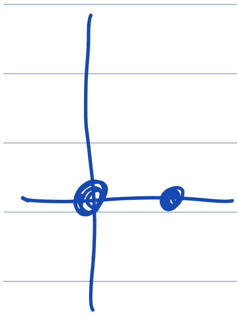
$x \ln x + x \ln\left(e^{-\frac{1}{x^2}}\right) =$

$= x \ln x + x\left(-\frac{1}{x^2}\right)$

$= x \ln x - \frac{1}{x}$ não existe limite quando $x \to 0$

3-a) $\longrightarrow$ trivial
(propriedades dos limites)

3-b) $f(x,0) = \dfrac{x}{\sqrt{x^2}} = \dfrac{x}{|x|}$



$$= \begin{cases} 1 & \text{se } x>0 \\ -1 & \text{se } x<0 \end{cases}$$

dois candidatos $\Longrightarrow$ não há limite.

3-c) basta analisar $\dfrac{x^2-y^2}{x^2+y^2}$ ← 2-e)

3-d) $\left| \dfrac{x^2}{\sqrt{x^2+y^2}} \right| = \dfrac{|x|}{\sqrt{x^2+y^2}} |x|$

$\dfrac{|x|}{\sqrt{x^2+y^2}} \leq 1$, $\quad |x| \to 0$

$\longrightarrow 0$

3-e)

$\left| x y^2 \operatorname{sen}\left(\frac{1}{y}\right) \right| \leq |x| y^2 \to 0$